complete

Purpose Complete a nonsquare polynomial matrix to a unimodular matrix

Syntax `[U,V] = complete(Q,[tol])`

Description If Q is a tall polynomial matrix then the command

```
[U,V] = complete(Q)
```

produces a unimodular matrix U of the form $U = [Q\ R]$. If Q is wide then the unimodular matrix U has the form $U = [Q;\ R]$. V is the inverse of U.

If Q does not have full rank or is not prime then no unimodular matrix U exists and an error message follows. Also if Q is square non-unimodular an error is reported.

The optional input argument tol is the tolerance used for the row or column reduction of Q that is part of the algorithm.

Example A tall polynomial matrix Q with column degrees 2 and 1 and dimensions 3×2 is generated by the command

```
Q = prand([2 1],3,2)

Q =

     1.6 - 1.1s - 0.026s^2    -1.1 + 0.75s
     0.5 - 0.52s - 0.56s^2    -0.75 + 0.93s
    -0.25 - 0.15s - 1.3s^2     0.31 + 2.7s
```

Q is completed to a unimodular matrix U by typing

```
[U,V] = complete(Q);

U

U =

     1.6 - 1.1s - 0.026s^2    -1.1 + 0.75s      0.2 + 0.00074s
     0.5 - 0.52s - 0.56s^2    -0.75 + 0.93s     0.4 + 0.016s
    -0.25 - 0.15s - 1.3s^2     0.31 + 2.7s      1 + 0.036s
```

It may be verified that U is unimodular and that V is its inverse by successively typing

```
det(U)

Constant polynomial matrix: 1-by-1

ans =
```



```
    -0.73
U*V
Constant polynomial matrix: 3-by-3
ans =
     1     0     0
     0     1     0
     0     0     1
```

Algorithm

Let Q be a full rank $n \times k$ polynomial matrix, with $n > k$. We wish to find an $n \times (n-k)$ polynomial matrix R such that $\lfloor Q \quad R \rfloor$ is unimodular. Let U be a unimodular matrix which reduces Q to the extended row-reduced form

$$UQ = \begin{bmatrix} Q_o \\ 0 \end{bmatrix}$$

If the $k \times k$ matrix Q_o is a constant matrix then it is nonsingular and the desired unimodular completion exists. Otherwise, the completion does not exist. The row reduction algorithm also yields the inverse $V = U^{-1}$ of U. Redefine

$$U := \begin{bmatrix} Q_o^{-1} & 0 \\ 0 & I \end{bmatrix} U, \quad V := V \begin{bmatrix} Q_o & 0 \\ 0 & I \end{bmatrix}$$

and partition $V = \begin{bmatrix} V_1 & V_2 \end{bmatrix}$. Then

$$UQ = \begin{bmatrix} I \\ 0 \end{bmatrix}, \quad UV_2 = \begin{bmatrix} 0 \\ I \end{bmatrix}$$

Hence, the desired completion is

$$\begin{bmatrix} Q & V_2 \end{bmatrix}$$

and its inverse is U.

If Q is not tall but wide than the algorithm is applied to the transpose of Q.

Diagnostics

The macro `complete` issues error messages if

- The input matrix is square non-unimodular
- The input matrix cannot be completed to a unimodular matrix because it is not prime
- The input matrix does not have full rank

See also

`colred, rowred` reduction to column or row reduced form