spherplot

Purpose Plot the value set of a polynomial family with spherical uncertainty set and independent uncertainty structure for a range of frequencies.
(A tool for robust stability testing via Zero Exclusion Condition).

Syntax

```
spherplot(p0,omega,r,W)
spherplot(p0,omega,r)
spherplot(p0,omega)
```

Description Graphical routine for plotting value sets of a *spherical polynomial family* for a range of frequencies. A family of polynomials $P = \{p(\cdot\,,\mathbf{q}) : \mathbf{q} \in Q\}$ is said to be spherical if $p(\cdot\,,\mathbf{q})$ has an independent uncertainty structure and the uncertainty set Q is an ellipsoid.

`spherplot(p0,omega,r,w)` plots the value sets for the spherical polynomial family, where `p0` is a nominal polynomial, `omega` is a vector of generalized frequencies, `r` is a robustness bound and `W` is a vector of diagonal entries of the weighting matrix. If the family has an independent uncertainty structure then the polynomial family can be expressed in the centered form

$$p(s,\mathbf{q}) = p_0(s) + \sum_{i=0}^{n} q_i s^i$$

where the weighted Euclidian norm of the vector of the uncertain parameters is bounded

$$\|\mathbf{q}\|_{2,W} \le r$$

`spherplot(p0,omega,r)`assumes that the weighting matrix `diag(W) = I`

`spherplot(p0,omega)` assumes that the weighting matrix `diag(W) = I` and the robustness margin `r = 1.` The vector of uncertain parameters is then bounded

$$\|\mathbf{q}\|_2 \le 1$$

As with other tools based on Zero Exclusion Condition, make sure that there is at least one stable member of the polynomial family. Also remember when you enter the `W` parameter, <u>you only assign the vector of diagonal entries</u>, not the whole matrix!

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Examples

Example 1

Consider the uncertain polynomial [1, pp.262]

$$p(s,q) = (0.5+q_0) + (1+q_1)s + (2+q_2)s^2 + (4+q_3)s^3$$

with the uncertainty bound $\|\mathbf{q}\|_{2,\mathbf{W}} \le 1$ and the weighting matrix $W = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$,

ie., $2q_0^2 + 5q_1^2 + 3q_2^2 + 1q_3^2 \le 1^2$. Note that this is what we can call abuse of notation since when assigning as an input parameter, the `W` parameter reads for a vector of diagonal entries of the weighting matrix.

Use the graphical method of Zero Exclusion Principle to test the robust stability of the given uncertain polynomial. First, let's transform the given polynomial into the centered form

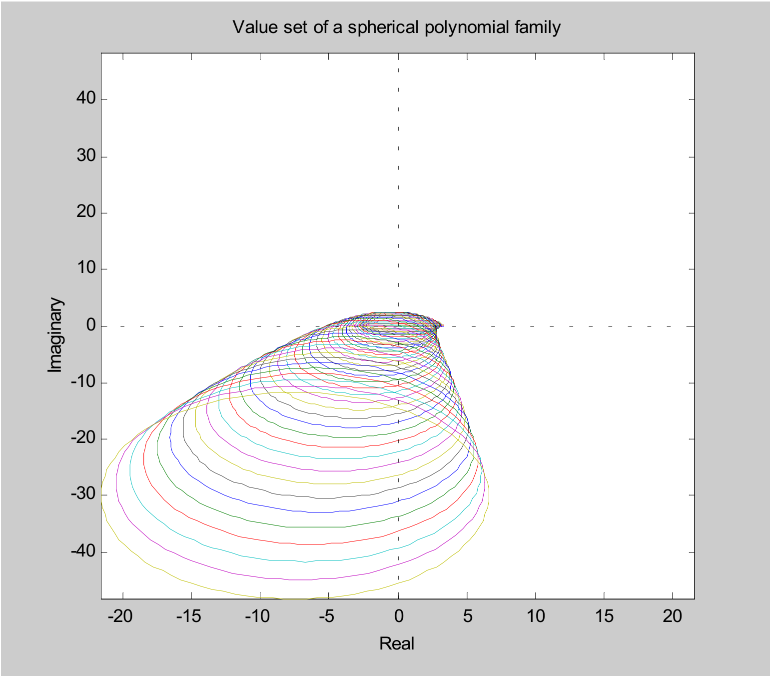
$$p(s,\mathbf{q}) = 0.5 + s + 2s^2 + 4s^3 + \sum_{i=0}^{3} q_i s^i$$

with the uncertainty bound unchanged. Now type

```
p0 = 0.5+s+2*s^2+4*s^3; W = [2,5,3,1]; r = 1; omega = 0:.05:2;
```

The graphical output is generated by

```
spherplot(p0,omega,r,W)
```



It can be seen that the Zero Exclusion Condition is violated and so we conclude that the given polynomial family is not robustly stable.

Example 2

Similarly to the previous example, test the following polynomial [1, pp.268] for robust stability

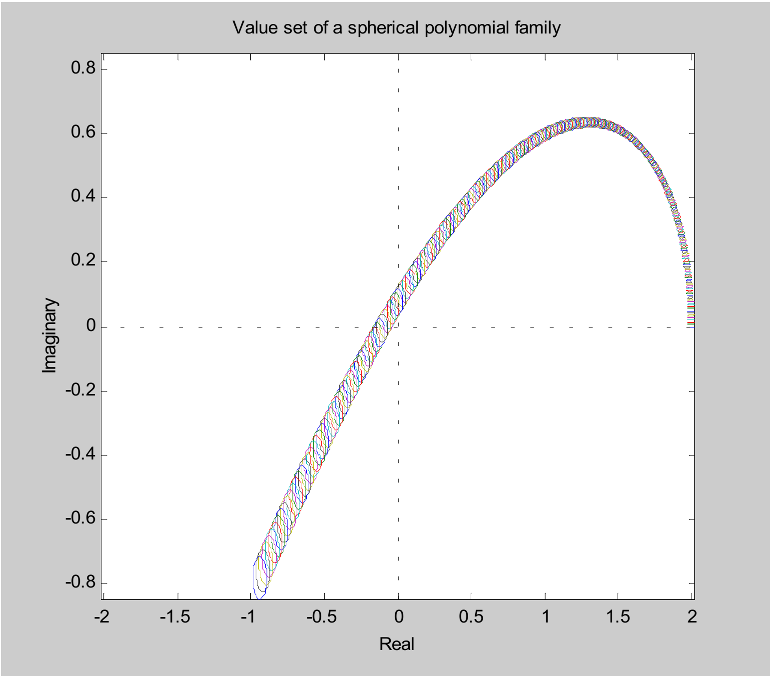
$$p(s,\mathbf{q}) = (2+q_0)+(1.4+q_1)s+(1.5+q_2)s^2+(1+q_3)s^3$$

with the uncertain parameters subject to

$$\|\mathbf{q}\|_2 \le 0.011$$

Then we type

```
p0 = 2+1.4*s+1.5*s^2+s^3;
r = 0.011;
omega = 0:0.005:1.4;
spherplot(p0,omega,r)
```



In this case the origin is excluded from the value set and we conclude that the polynomial family is robustly stable.

See also

`khplot`	Value set for an interval polynomial.
`ptopplot`	Value set for a polytope of polynomials.
`vsetplot`	Value set for polynomials with general uncertainty structure.

References

[1] R. Barmish: *New Tools for Robustness of Linear Systems*. Macmillan Publishing Company. New York, 1994.